**Discrete Distributions**

**Definitions:**

1. **Geometric Distribution**:
   * Describes the probability of the number of trials needed to get the first success in a series of independent and identical Bernoulli trials (e.g., flipping a coin).
   * **Example**: The number of coin tosses until you get the first heads.
   * **Probability Mass Function (PMF)**: P(X=k)=(1−p)k−1pP(X = k) = (1 - p)^{k-1} pP(X=k)=(1−p)k−1p where ppp is the probability of success and kkk is the number of trials.
2. **Binomial Distribution**:
   * Models the number of successes in a fixed number of independent Bernoulli trials, where each trial has the same probability of success.
   * **Example**: The number of heads in 10 coin tosses.
   * **Probability Mass Function (PMF)**: P(X=k)=(nk)pk(1−p)n−kP(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}P(X=k)=(kn​)pk(1−p)n−k where nnn is the number of trials, kkk is the number of successes, and ppp is the probability of success in each trial.
3. **Poisson Distribution**:
   * Describes the probability of a given number of events happening in a fixed interval of time or space, provided that these events occur with a known average rate and independently of each other.
   * **Example**: The number of cars passing a checkpoint in an hour.
   * **Probability Mass Function (PMF)**: P(X=k)=λke−λk!P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}P(X=k)=k!λke−λ​ where λ\lambdaλ is the average rate of events per interval and kkk is the number of events.

**Key Differences:**

1. **Nature of Trials**:
   * **Geometric**: Focuses on the number of trials until the first success.
   * **Binomial**: Focuses on the number of successes in a fixed number of trials.
   * **Poisson**: Focuses on the number of events occurring in a fixed interval (time/space).
2. **Number of Trials**:
   * **Geometric**: Variable (until the first success).
   * **Binomial**: Fixed number of trials.
   * **Poisson**: No explicit "trials," only the occurrence of events over time/space.
3. **Parameter Involvement**:
   * **Geometric**: Only one parameter ppp (probability of success).
   * **Binomial**: Two parameters nnn (number of trials) and ppp (probability of success).
   * **Poisson**: Only one parameter λ\lambdaλ (rate of event occurrence).
4. **Distribution Type**:
   * **Geometric and Binomial**: Deal with discrete Bernoulli trials (binary outcomes).
   * **Poisson**: Deals with counts of events in continuous time or space intervals.
5. **Typical Use**:
   * **Geometric**: When you're interested in how long it takes for a success to occur.
   * **Binomial**: When you're interested in the total number of successes in a fixed number of trials.
   * **Poisson**: When you're modeling the number of events happening over a continuous interval.

These distributions are all part of discrete probability distributions, but they apply to different types of random processes and scenarios.